Section 6.4 Solutions

#1 - 11: Write the equation in logarithmic form.

1) $3^2 = 9$ (write the word log, make the 3 the base, switch the 2 and 9)

Answer: $\log_3 9 = 2$

3) $3^4 = 81$ (write the word log, make the 3 the base, switch the 4 and 81)

Answer: $\log_3 81 = 4$

5) $3^{-1} = \frac{1}{3}$ (write the word log, make the 3 the base, switch the -1 and 1/3)

Answer: $\boldsymbol{log_3 \frac{1}{3} = -1}$

7) $e^y = x$ (write the word log, make the e the base, switch the y and x, then rewrite with ln notation)
$\log_e x = y$

Answer: ln(x) = y

9) $e^3 = 20.09$ (write the word log, make the e the base, switch the 3 and 20.09, then rewrite with ln notation)

$\text{Log}_e 20.09 = 3$

Answer: ln(20.09) = 3

11) $e^1 = 2.72$ (write the word log, make the e the base, switch the 1 and 2.72, then rewrite with ln notation)

$\text{Log}_e 2.72 = 1$

Answer: ln(2.72) = 1

#12 - 26: Write the equation in exponential form.

13) $\log_3 81 = 4$ (scratch out log, leave three low, switch 81 and 4)

Answer: $3^4 = 81$

15) $\log_2 64 = 6$ (scratch out log, leave e low, switch 64 and 6)

Asnwer: $2^6 = 64$

17) $\log_6 6=1$ (scratch out log, leave 6 low, switch 6 and 1)

Answer: $6^1 = 6$

19) $\log x=3$ (write with base 10, then scratch out log, leave 10 low, switch x and 3)

$\text{Log}_{10} x = 3$

Answer: $10^3 = x$

21) $\ln(x) = 1$ (write with base e, then scratch out log leave e low, switch 1 and x)

$\text{Log}_e x = 1$

Answer: $e^1 = x$

23) $\ln(2x) = w$ (write with base e, then scratch out log leave e low, switch 2x and w)

$\text{Log}_e 2x = w$

Answer: $e^w = 2x$

25) $\ln(e^2) = 2$ (write with base e, then scratch out log leave e low, switch e^2 and 2)

$\text{Log}_e(e^2) = 2$

Answer: $e^2 = e^2$

26) $\ln(e^3) = 3$ (write with base e, then scratch out log leave e low, switch e^3 and 3)

$\text{Log}_e(e^3) = 3$

Answer: $e^3 = e^3$

#27 - 56: Find the logarithm value without using a calculator.

27) $\log_2 2$ (asks 2 to what power is 2, $2^x = 2$)

Answer: 1

29) $\log_7 7$ (asks 7 to what power is 7, $7^x = 7$)

Answer: 1

31) $\log_3 1$ (asks 3 to what power is 1, $3^x = 1$)

Answer: 0

33) $\log_4 1$ (asks 4 to what power is 1, $4^x = 1$)

Answer: 0

35) $\log_4 64$ (asks 4 to what power is 64, $4^x = 64$)

Answer: 3

37) $\log_5 5$ (asks 5 to what power is 5, $5^x = 5$)

Answer: 1

39) $\log 1$ (asks 10 to what power is 1, $10^x = 1$)

Answer: 0

41) $\log 100$ (asks 10 to what power is 100, $10^x = 100$)

Answer: 2

43) $\log_2 128$ (asks 2 to what power is 128, $2^x = 128$)

Answer: 7

45) $\log_2 2^3$ (asks 2 to what power is 2^3, $2^x = 2^3$)

Answer: 3

47) $\log_5 5^6$ (asks 5 to what power is 5^6, $5^x = 5^6$)

Answer: 6

49) $\log_4 4^5$ (asks 4 to what power is 4^5, $4^x = 4^5$)

Answer: 5

51) ln(e) (asks e to what power is e, $e^x = e$)

Answer: 1

53) $\ln(e^3)$ (asks e to what power is e^3, $e^x = e^3$)

Answer: 3

55) $\ln(e^{-2})$ (asks e to what power is e^{-2}, $e^x = e^{-2}$)

Answer: -2

Section 6.4 Logarithmic Functions

#57 - 65: Use a calculator to approximate the logarithms. Round to 4 decimal places.

57) log 6 (I enter the problem exactly as it appears)

Answer: .7782

59) $log\frac{1}{3}$ (I enter log(1/3) on my calculator)

Answer:-.4771

61) $\log(5^{-3})$ (I enter log(5^-3) on my calculator)

Answer:-2.0969

63) ln(7) (I type the problem as it appears)

Answer: 1.9459

65) $\ln(2^4)$ (I enter Ln(2^4) on my calculator)

Answer: 2.7726

#66-71 (graphs of common logarithmic functions)

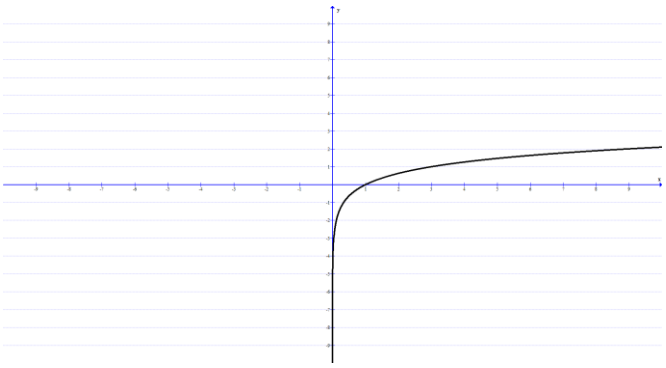
67) $y = \log_3 x$

a) Graph the logarithmic functions. First write the equation in exponential form, then create a table of values and plot the points.

Write without log $3^y = x$

Create a table of values, I will put the numbers 2,1,0,-1,-2 in the y column and solve for x.

x	y	point
3^2	2	(9,2)
3^1	1	(3,1)
3^0	0	(1,0)
3^{-1}	-1	(1/3, -1)
3^{-2}	-2	(1/9, -2)



b) State the domain of each function.

The graph is supposed to always be to the right of the y-axis. The graph is never supposed to touch the y-axis, so the domain should be x >0

Answer: $(\mathbf{0}, \infty)$

69) y = ln(x)

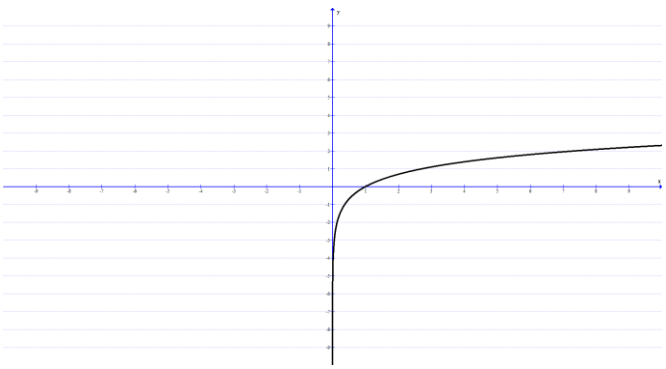
a) Graph the logarithmic functions. First write the equation in exponential form, then create a table of values and plot the points.

$y = \log_e x$ (write so can see the base)

$e^y = x$ (write without log)

Create a table of values, I will put the numbers 2,1,0,-1,-2 in the y column and solve for x.

x	y	point
e^2	2	(7.39, 2)
e^1	1	(2.72, 1)
e^0	0	(1, 0)
e^{-1}	-1	(.37, -1)
e^{-2}	-2	(.14, -2)



b) State the domain of each function.

The graph is supposed to always be to the right of the y-axis. The graph is never supposed to touch the y-axis, so the domain should be x >0

Answer: $(\mathbf{0}, \infty)$

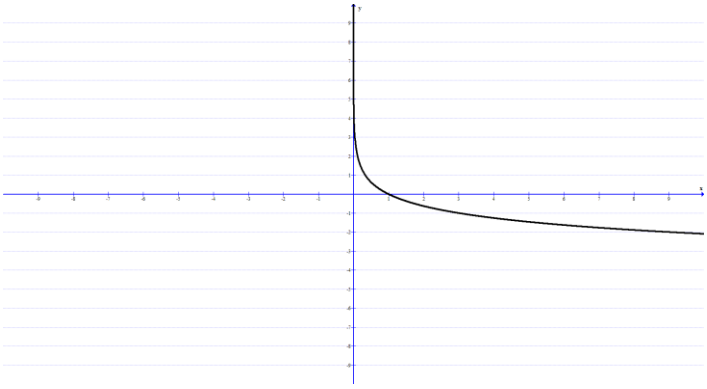
71) $y = log_{1/3}x$

a) Graph the logarithmic functions. First write the equation in exponential form, then create a table of values and plot the points.

$\left(\frac{1}{3}\right)^{y} = x$

Create a table of values, I will put the numbers 2,1,0,-1,-2 in the y column and solve for x.

x	y	point
$\left(\frac{1}{3}\right)^{2}$	2	(1/9, 2)
$\left(\frac{1}{3}\right)^{1}$	1	(1/3, 1)
$\left(\frac{1}{3}\right)^{0}$	0	(1, 0)
$\left(\frac{1}{3}\right)^{-1}$	-1	(3, -1)
$\left(\frac{1}{3}\right)^{-2}$	-2	(9,-2)



b) State the domain of each function.

The graph is supposed to always be to the right of the y-axis. The graph is never supposed to touch the y-axis, so the domain should be x >0

Answer: $(\mathbf{0}, \infty)$

#72- 84:

73a) f(x + 1)
The x + 1 needs to go in the parenthesis.

Answer #73a: f(x + 1) = $\log_2(x+1)$

73b) To find the domain of a logarithm just set the argument > 0

x + 1 > 0
x > - 1

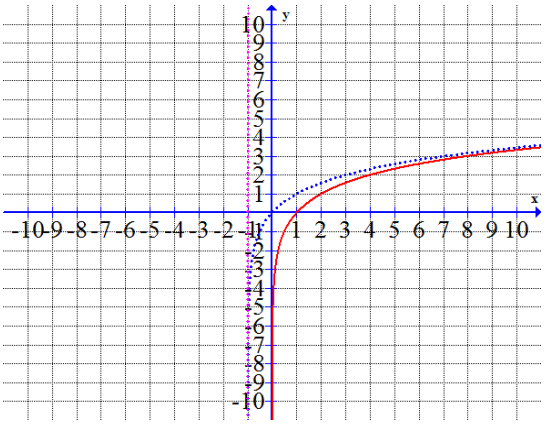
Answer #73b: x > -1 or $(-1, \infty)$

73c) The +1 in the parenthesis shifts the graph to the left 1 unit.

Answer #73c: Shifts left 1

73d) Just shift each point in the graph of f(x) one unit to the left. I showed the x>-1 domain as a vertical asymptote drawn in purple. The graph will not exist to the left of this vertical line x = -1.

Graph of f(x + 1) drawn in blue



75a) f(x – 2)
The x – 3 needs to go in the parenthesis.

Answer #75a: f(x – 2) = $\log_2(x - 2)$

75b) To find the domain of a logarithm just set the argument > 0

x – 2 > 0
x > 2

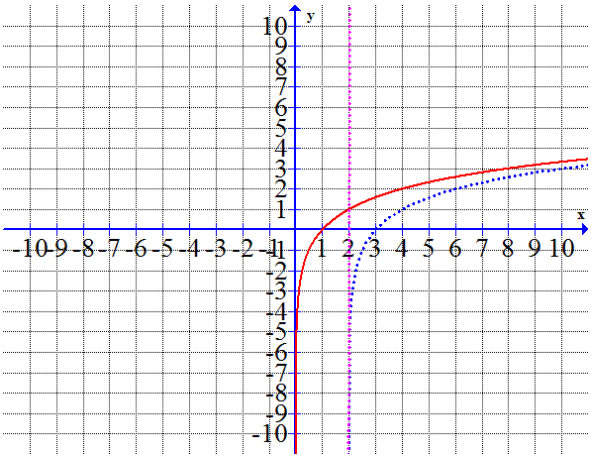
Answer #75b: x > 2 or $(2, \infty)$

75c) The (-2) in the parenthesis shifts the graph to the right 3 units.

Answer #75c: Shifts right 2

75d) Just shift each point in the graph of f(x) two units to the right. I showed the x>2 domain as a vertical asymptote drawn in purple. The graph will not exist to the left of this vertical line x = 2.

Graph of f(x – 2) drawn in blue



77a) f(x) + 2

Just add 2 to the function. The 2 should not go in the parenthesis as it is not inside the parenthesis.

Answer #77a: $f(x) + 2 = \log_2(x) + 2$

77b) set the argument greater than 0 to find the domain. The 2 is not inside the parenthesis, so it does not affect the domain.

(argument > 0 gives-) $x > 0$

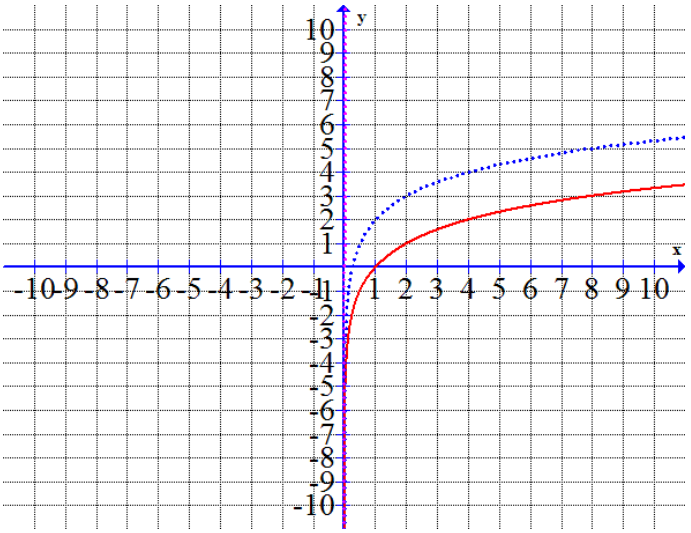
Answer #77b: $x > 0$ or $(0, \infty)$

77c) The plus 2 will shift the graph up 2 units.

Answer #77c: shift up 2 units

77d) Just shift each point in the graph of f(x) two units to the up. I showed the x>0 domain as a vertical asymptote drawn in purple. The graph will not exist to the left of this vertical line x = 0.

Graph of f(x) + 2 drawn in blue



79a) f(x) - 2
Just subtract 2 from the function. The 2 should not go in the parenthesis as it is not inside the parenthesis.

Answer #79a: $f(x) - 2 = \log_2(x) - 2$

79b) set the argument greater than 0 to find the domain. The 2 is not inside the parenthesis, so it does not affect the domain.

(argument > 0 gives-) $x > 0$

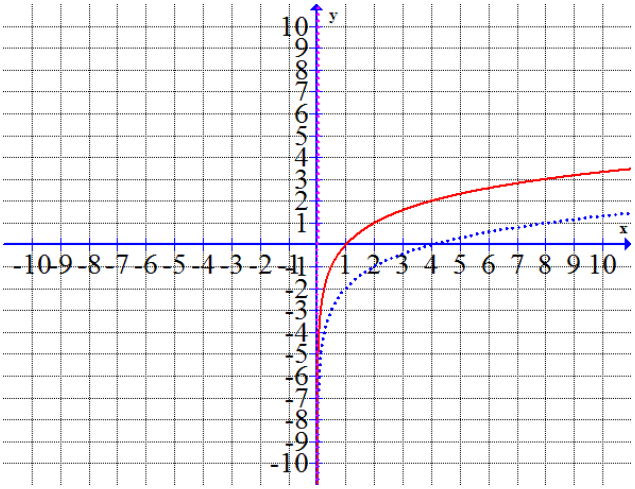
Answer #79b: $x > 0$ or $(0, \infty)$

79c) The minus 2 will shift the graph down 2 units.

Answer #79c: shift down 2 units

79d) Just shift each point in the graph of f(x) two units to the down. I showed the x>0 domain as a vertical asymptote drawn in purple. The graph will not exist to the left of this vertical line x = 0 which is the y-axis.

Graph of f(x) - 2 drawn in blue



81a) To find f(-x) the –x will go inside the parenthesis.

Answer #81a: $f(-x) = \log_2(-x)$

81b) set the argument greater than 0 to find the domain.

$-x > 0$

$+x \quad +x$

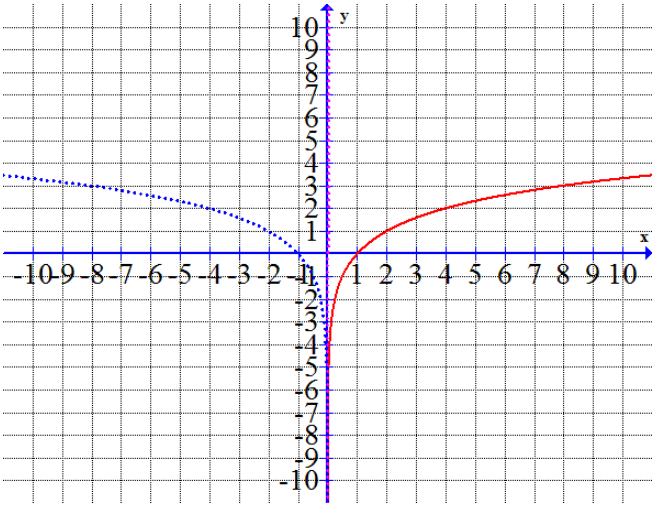
$0 > x$

Answer #81b: domain 0 > x, this is the same as x < 0 which is the same as $(-\infty, 0)$ any one of these three answers is correct.

81c) Since the negative is inside the parenthesis, the graph gets reflected over the y-axis.

Answer #81c: reflects over y-axis

81d) Just reflect each point over the y-axis. The graph will now only exist to the left of the y-axis. The vertical asymptote will still be at x = 0 (or the y-axis). It's now the right edge of the graph as opposed to the left edge of the graph.



83a) Place the 3 to the left of the log to find this function. The 3 does not go inside the parenthesis as it is not inside the parenthesis.

Answer #83a: $3f(x) = 3\log_2(x)$

83b) set the argument greater than 0 to find the domain. The 3 is not inside the parenthesis, so it does not affect the domain.

(argument > 0 gives-) $x > 0$

Answer #83b: $x > 0$ or $(0, \infty)$

83c) The y-value of each point of the original graph will get multiplied by 3 when I create my new graph. Each y-value will be further away from the x-axis than in the graph of f(x). This is why we say the graph gets stretched.

Answer #83c: Stretched.

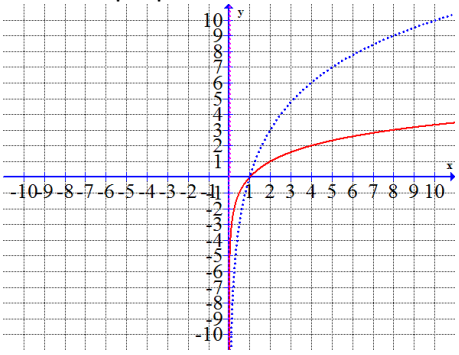
83d) This is a non-rigid transformation. I need to make a table of values to sketch an accurate graph. We can use the x's from the given table. We create y's by multiplying each y-value by 2.

The y-value gets pulled away from the y-axis when we multiply the y-values by 3. This is why we say the function is stretched.

The vertical asymptote doesn't move as the graph isn't shifted left or right.

The vertical asymptote is drawn in purple.

3f(x) is drawn in blue. The vertical asymptote is drawn in purple.



Here are the points that are marked in the original graph

x	f(x)
.25	-2
.5	-1
1	0
2	1
4	2

The table for 3f(x) will have the same x-values, but the y's will be multiplied by 3.

Here is the table for 3f(x)

x	2f(x)
.25	-6
.5	-3
1	0
2	3

	4	6